Trajectories of Mathematics Performance: From Preschool to Postsecondary

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Mathematics

Successful performance in mathematics (i.e., math) requires an understanding of numbers, the quantities represented by numbers, counting, and comparison of amounts. Math also requires an understanding of the concepts of addition, subtraction, multiplication, and division and algorithms for quickly solving such problems. Students must be able to apply their calculation and computation skills to math problems featuring fractions, decimals, percentages, measurement, and algebra. Additionally, students must be familiar with geometric shapes and concepts, as well as positive and negative numbers. Students begin learning math informally as babies and toddlers, and as students learn more about math as they age, these math skills set the stage for later success with math.

Math performance is directly related to employment opportunities in adulthood (Murnane, Willett, Braatz, & Duhaldeborde, 2001), and math outcomes are as important as reading outcomes for success in school. For these reasons, it is necessary to understand how early in a student's school career educators can identify students who struggle with math in order to provide proper instruction and support. Without identification and support, students may continue to struggle with math throughout middle school and high school. Additionally, difficulty with math may influence college decisions and workforce placement.

In this brief, we collected research literature on the math performance of students in preschool, elementary, middle, and high school. In each study, researchers measured math performance at one time point and then measured math performance at a subsequent time point or time points at least one year later. This collection of research provides a snapshot of the math trajectories of school-age students and may have implications for policy and practice.

Literature Search

We reviewed research studies published from January 1985 to July 2015 that focused on math-related predictors of math achievement across one or more grade levels. The review included a computerized search of the literature using scholarly databases with combinations of the following search terms: *math, growth, longitudinal, predictor,* and *trajectory*. We included studies that met the following inclusion criteria:

(a) the focus of the study was to examine a typical longitudinal trajectory of math achievement for participants between infancy and college graduation,

(b) the study included at least one measure of math achievement or adult outcomes (e.g., college enrollment rates),

(c) the study included appropriate results to determine the longitudinal predictors of math achievement, and

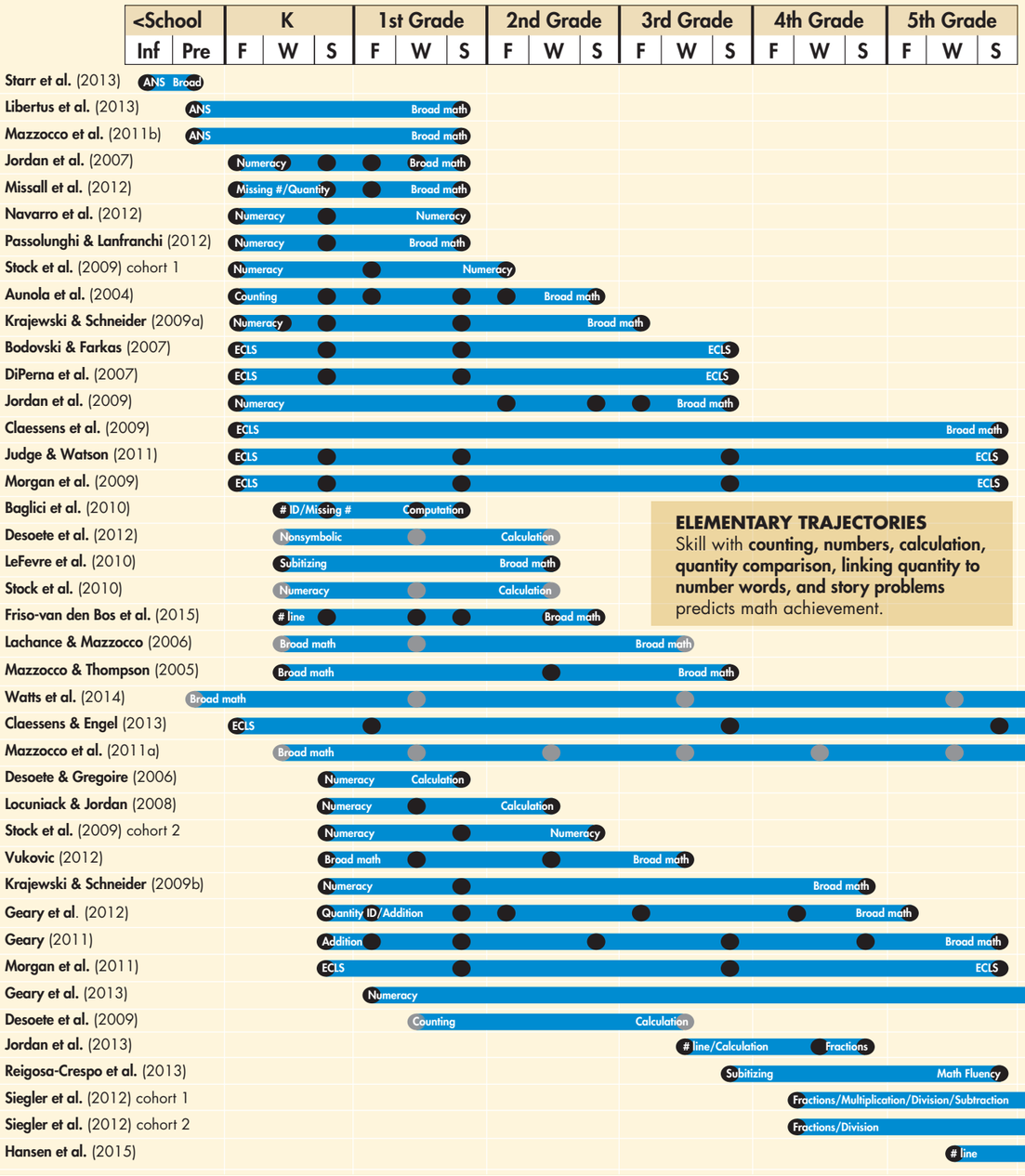
(d) the study was published in English in a peer-reviewed journal.

Coding of Studies

We coded 46 studies (two of which had multiple cohorts for a total of 48 learning trajectories) that met inclusion for the following information: year of publication, location, sample size, attrition, average age (or grade) of participants at the start of the study, data collection time points, measures of math achievement, and results. For time points, we coded each data point (e.g., time 1, time 2, time 3) and the time of year data were collected (i.e., fall, winter, spring). With regard to measures of math achievement, we recorded the measures administered at each time point, a short description of the measure and the skills assessed, and the reliability and validity of the measures. Finally, for each study, we recorded specific results and significance levels of the results.

Prepared by: Gena Nelson, University of Minnesota, and Sarah R. Powell, University of Texas at Austin

From Preschool through the Elementary Grades



ELEMENTARY TRAJECTORIES
Skill with **counting, numbers, calculation, quantity comparison, linking quantity to number words, and story problems** predicts math achievement.

Definitions

ANS: Approximate Number System (estimation skill with quantity)
Broad math: Measure assesses a variety of math skills
Calculation: Skill with addition, subtraction, multiplication and/or division; may be called "math facts"
Computation: Skill with calculation in multiple steps (e.g., multi-digit addition; long division)
ECLS: Early Childhood Longitudinal Study (United States data set)
ID: identification
Missing #: Identification of the missing number in a sequence (e.g., 4, 5, _ , 7)
Nonsymbolic: Math with pictures, not numbers
Number line: Skill with placing numbers on a number line
Numeracy: Understanding of counting and comparing
Quantity: How many in a set: Instantly knowing how many (e.g., know *** is "three")

TIME LINE

We present a visual time line of the included studies. All studies indicated that math performance data collected at time 1 predicted math performance at later time points. We placed dots on the blue line to indicate each time data was collected from students.

- Black dots indicate exact time (fall, winter, spring) of test.
- Gray dots indicate estimated time of test. When authors did not provide exact time of test, we placed the time of test in the winter of the school year.

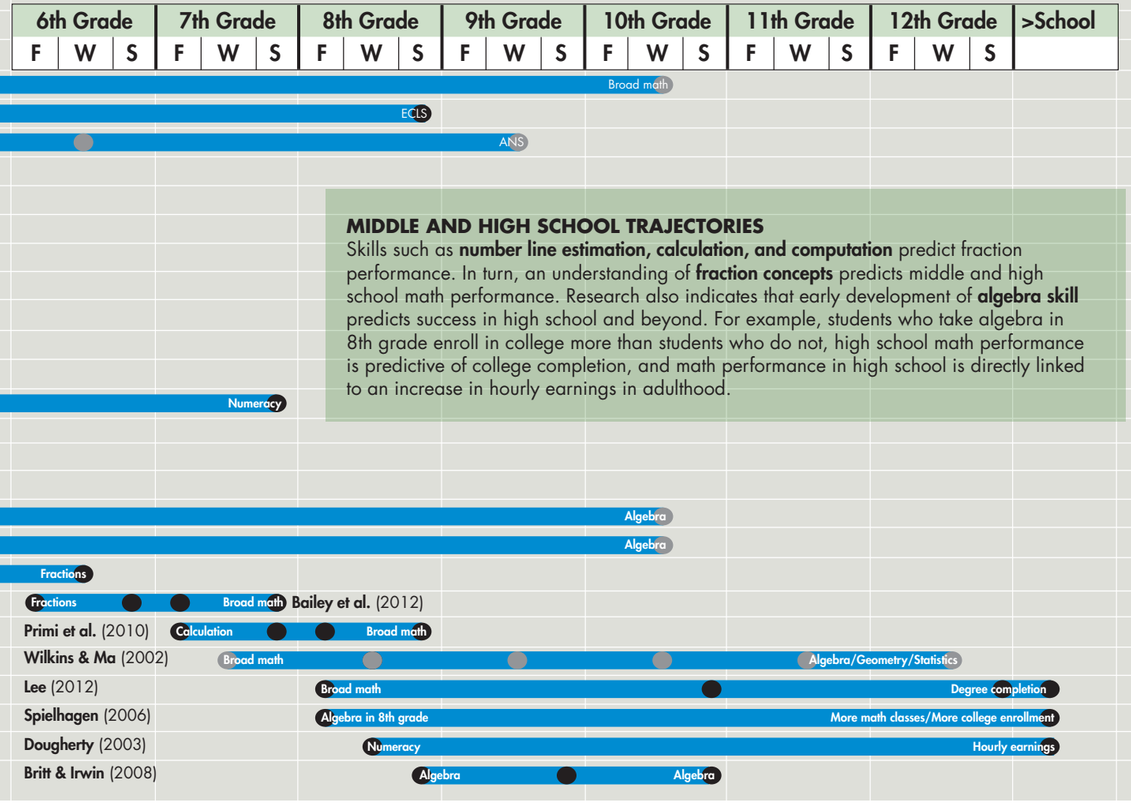
We also provide a brief description of the math measure(s) administered at the first and last data points.

SUMMARY OF STUDIES

The 46 studies were published between 2003 and 2015 (M = 2010), and data for the studies were collected in 11 different countries. The majority (65%) of studies collected data in the United States, 29% were conducted in Europe, and 6% were conducted in other countries. The average sample size across studies was 2,059 participants (range = 17 to 13,043). Authors reported attrition rates (i.e., students leaving the study) in 27 studies (M = 22%, range = 0 to 48.1%); although the attrition rates were high in some studies, this was expected due to the longitudinal nature of the data.

Regarding the time of the first data point, the majority of studies (63%) collected the first data point during participants' kindergarten year of school. Other studies collected the first data point in elementary grades (i.e., Grades 1-5; 15%), middle and high school (Grades 6-12; 15%), and prior to kindergarten entry (8%). The number of data points collected in each study ranged from 2 to 8 (M = 3.67), and the majority of studies (76%) collected data that included more than two grade levels (e.g., first, second, third grades). On average, studies collected data across 4 grades (SD = 2.16).

From Middle School to Postsecondary



MIDDLE AND HIGH SCHOOL TRAJECTORIES
Skills such as **number line estimation, calculation, and computation** predict fraction performance. In turn, an understanding of **fraction concepts** predicts middle and high school math performance. Research also indicates that early development of **algebra skill** predicts success in high school and beyond. For example, students who take algebra in 8th grade enroll in college more than students who do not, high school math performance is predictive of college completion, and math performance in high school is directly linked to an increase in hourly earnings in adulthood.

Implications for Policy and Practice

From this review of the research on math trajectories, we see several trends that have implications for policy and practice:

Math performance at earlier time points is predictive of math performance at later time points.

With this knowledge, educators should use measures of math performance to identify students who have difficulty with math. If students with math difficulty receive quality math instruction in problem areas, then math performance is likely to improve within the grade level and across grade levels. Additionally, educators should understand which math skills are necessary for later math success. For example, development of numeracy skills is closely linked to later computation skill. Similarly, strong fraction skill is linked to better algebra performance.

Students who perform well in math continue to perform well in math; students who experience difficulty with math skills or concepts continue to experience difficulty.

When authors disaggregated data by difficulty or disability category, students with difficulty continued to demonstrate low math performance across grade levels. Educators must identify these students early and provide help in problem areas. Educators must be provided with the knowledge to teach math effectively and efficiently, and educators must know how to provide instruction on foundational math skills.

Math performance in school has implications for students in adulthood.

Most students are in school for 13 to 15 years. During this time, educators must improve the math understanding and performance of students, because success with school math relates to success as an adult. To improve the college access for students, educators must provide a strong math curriculum for all students. In the same way, to improve adulthood earnings, students must receive adequate math instruction in preschool, elementary, and middle school that enables access to higher-level math classes in high school.

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